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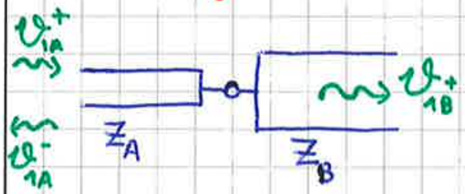
C21 - Lignes en cascades

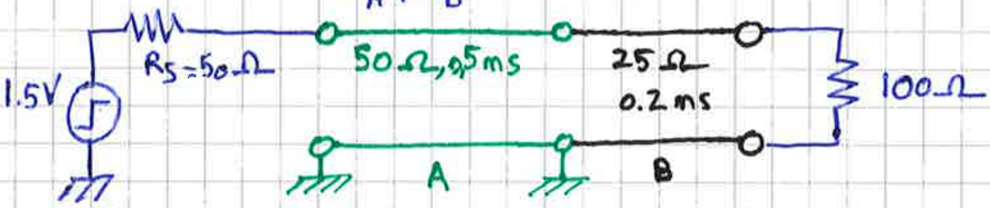


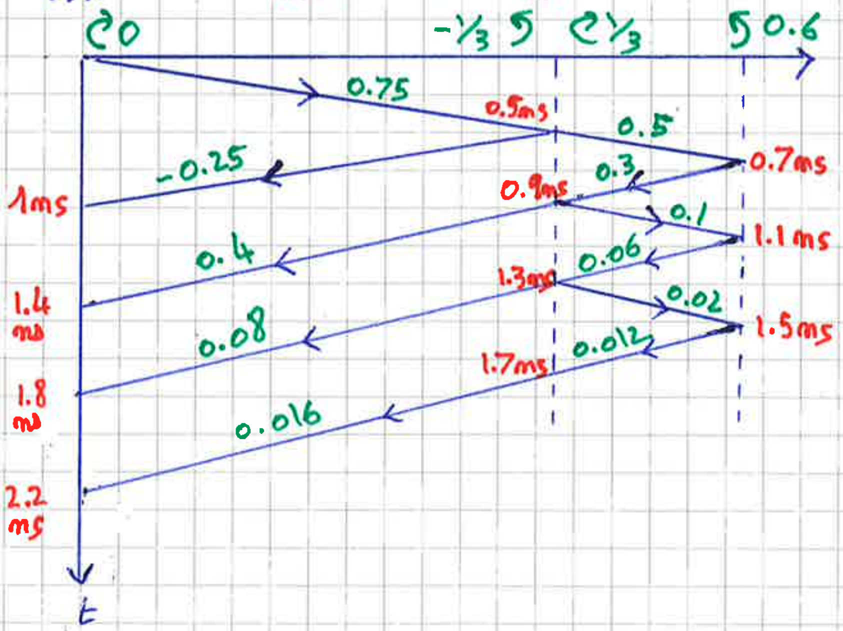
$$\begin{cases} v_{1A}^- = r_{AB}\, v_{1A}^+ \\ v_{1B}^+ = T_{AB}\, v_{1A}^+ \end{cases}$$

$$\boxed{r_{AB} = \frac{Z_B - Z_A}{Z_B + Z_A}} \quad , \quad T_{AB} = ?$$

CL : $v_{1A}^+ + v_{1A}^- = v_{1B}^+ \Rightarrow \boxed{1 + r_{AB} = T_{AB}}$

Attention : $r_{BA} = \dfrac{Z_A - Z_B}{Z_A + Z_B} = -r_{AB}$, $T_{BA} = 1 + r_{BA} \neq T_{AB}$



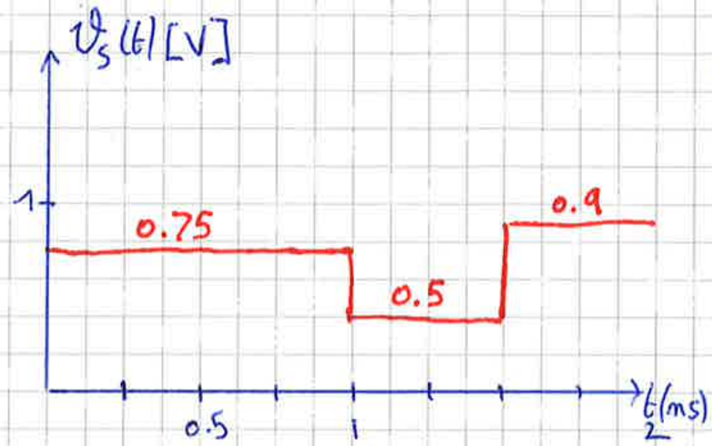


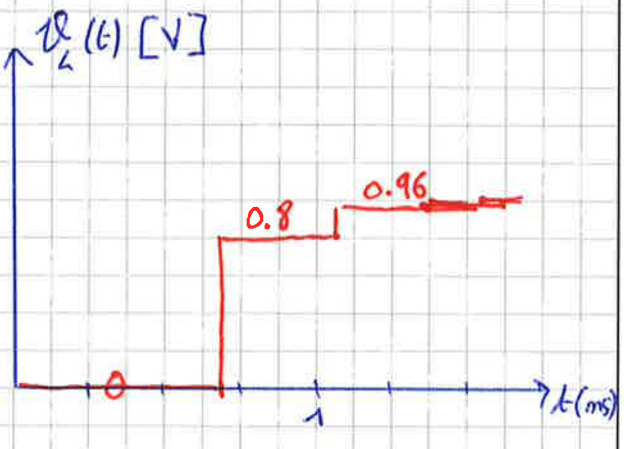
$$r_{AB} = \frac{25-50}{25+50} = -1/3$$

$$r_{BA} = +1/3$$

$$r_L = \frac{75}{125} = 0.6$$

$$v_1^+ = \frac{1.5 \cdot 50}{50+50} = 0.75\text{ V}$$





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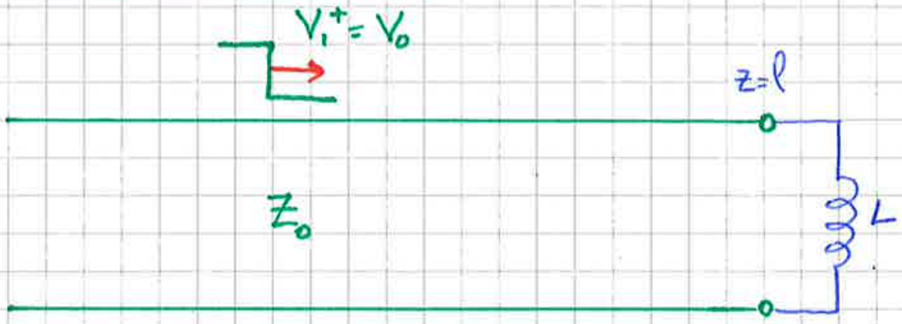
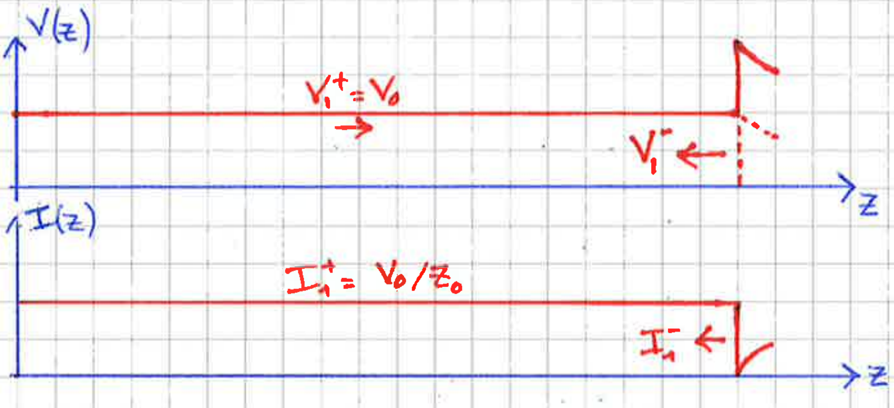
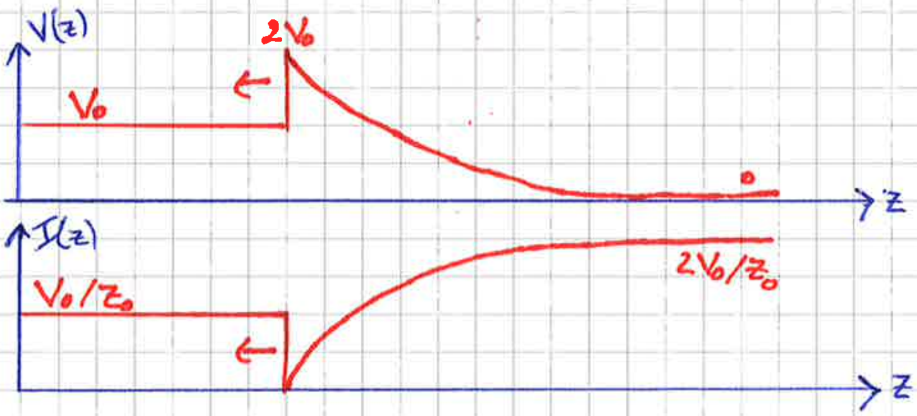
e) Terminaisons réactives





immédiatement après la réflection

= circuit "ouvert"



plus tard

= circuit "fermé"

C23 – $V_L(t) = L \frac{d}{dt}\left(I_L(t)\right)$

$$\Rightarrow \quad v_1^+(l,t) + \underbrace{v_1^-(l,t)}_{\text{inconnue}} = L \frac{d}{dt}\left\{\frac{1}{Z_0}\left(v_1^+(l,t) - v_1^-(l,t)\right)\right\}$$

$$\Rightarrow \quad \frac{d v_1^-(l,t)}{dt} + \frac{Z_0}{L} v_1^-(l,t) = \frac{d v_1^+}{dt} - \frac{Z_0}{L} v_1^+(l,t)$$

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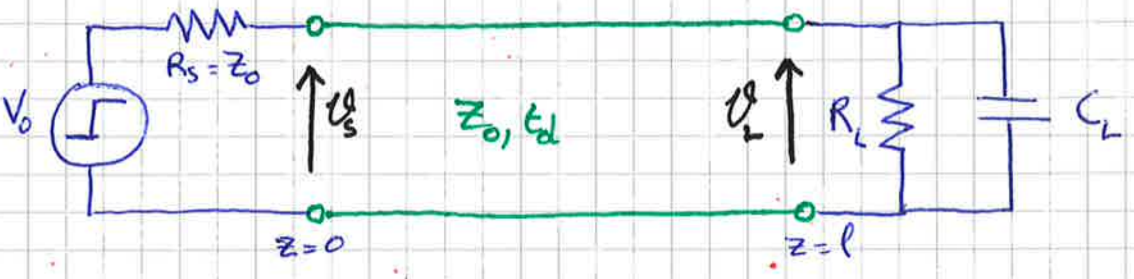
solution : $v_1^-(\ell,t) = -V_0 + K e^{-Z_0 t/L}$

constante K ? $\quad I_L(t=0) = 0$

$$v_1^-(\ell,t)\big|_{t=0} = V_0 \Rightarrow K = 2V_0$$

C25_ Charge capacitive et résistive



Pour $t \gg t_d$
$$\begin{cases} v_L(t) = v_1^+(\ell,t) + v_1^-(\ell,t) \\ I_L(t) = \dfrac{v_1^+(\ell,t)}{Z_0} - \dfrac{v_1^-(\ell,t)}{Z_0} \end{cases}$$

et $$I_L(t) = \frac{v_L(t)}{R_L} + C_L \frac{d v_L(t)}{dt} \Rightarrow \frac{d v_L}{dt} + \frac{1}{R_L C_L} v_L = \frac{I_L}{C_L}$$

$$\left(\Rightarrow \overbrace{\frac{dV_1^+}{dt}}^{=0} + \frac{dV_1^-}{dt} + \frac{V_1^+}{R_L C_L} + \frac{V_1^-}{R_L C_L} = \frac{V_1^+}{Z_0 C_L} - \frac{V_1^-}{Z_0 C_L} \Rightarrow \right)$$

$$\Rightarrow \frac{dV_1^-}{dt} + \frac{R_L + Z_0}{R_L Z_0 C_L} V_1^- = \frac{R_L - Z_0}{R_L Z_0 C_L} V_1^+$$

$$\Rightarrow V_1^-(\ell,t) = \frac{R_L - Z_0}{R_L + Z_0} V_1^+ + \text{cste}\, \exp\left(-\frac{R_L + Z_0}{R_L Z_0 C_L}(t - t_d)\right)$$

cste ? $\quad t \to t_d \Rightarrow v_L \to 0 \quad \left(\text{car } i_c = C \dfrac{du_c}{dt} \right)$

$\Rightarrow V_1^- \to -V_1^+$

la tension aux bornes d'une capa est toujours dérivable donc continue: comme elle valait 0 avant td, elle vaut encore 0 à td

$$\Rightarrow \text{cste} = -V_1^+ - \frac{R_L - Z_0}{R_L + Z_0} V_1^+ = V_1^+ \left(\frac{-R_L - Z_0 - R_L + Z_0}{R_L + Z_0} \right) = -\frac{2R_L V_1^+}{R_L + Z_0}$$

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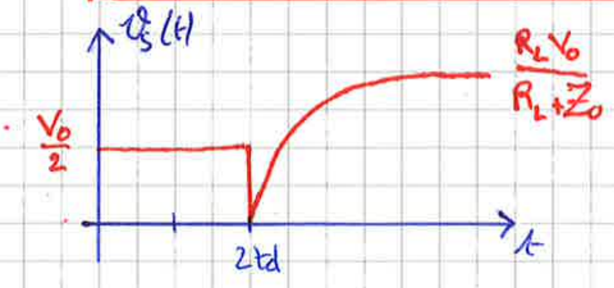
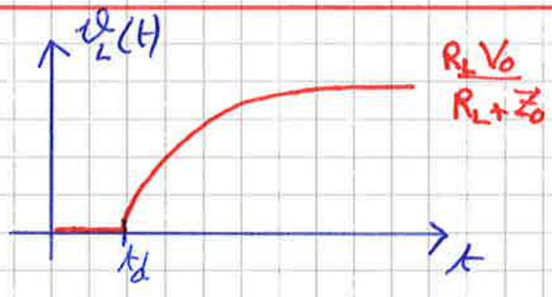
$$\Rightarrow \boxed{V_1^-(\ell,t) = \underbrace{V_1^+}_{V_0/2} \left(\frac{R_L - Z_0}{R_L + Z_0} - \frac{2R_L}{R_L + Z_0} e^{-[(R_L+Z_0)/(R_L Z_0 C_L)](t-t_d)} \right)}$$





$$V_1^-(\ell, t = t_d) = -\frac{V_0}{2}$$

$$V_1^-(\ell, t \to +\infty) = \frac{V_0}{2} \frac{R_L - Z_0}{R_L + Z_0}$$

$$V_S(t) = \begin{cases} \frac{V_0}{2} & (t \leq 2t_d) \\ \frac{V_0}{2} + V_1^-(\ell, t - t_d) & t > t_d \times 2 \end{cases}$$

$$V_L(t) = \begin{cases} \frac{V_0}{2} + V_1^-(\ell, t) & t > t_d \\ 0 & t \leq t_d \end{cases}$$